Calculation of Compressible Turbulent Boundary Layers with Heat and Mass Transfer

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In this paper we present a general method for calculating turbulent boundary layers in two-dimensional flows and investigate its accuracy for compressible flows with heat and mass transfer. The method is based on the ideas of eddy transport coefficients and the numerical solution of the governing equations in differential form. The experimental data considered cover a Mach number range of 0 to 6.7 and include flows with and without pressure gradients. The results indicate good agreement at high Reynolds numbers. At low Reynolds numbers the agreement is not as good, and further work needs to be done in such cases.

Nomenclature

c_f = local skin-friction coefficient, see Eq. (32)
c_{f0} = local skin-friction coefficient with no mass transfer
$\bar{c}_f$ = average skin-friction coefficient
F = mass transfer parameter $(\rho v)_w/\rho_e u_e$
H = total enthalpy, $h + u^2/2$
l = mixing length
L = reference length
M = Mach number
p = pressure
p^+ = pressure-gradient parameter, $[\nu_e u_e(du_e/dx)/(u_w^*)^3]$
Pr = molecular Prandtl number
Pr_t = turbulent Prandtl number
$\dot{q}$ = heat flux
R_x = Reynolds number, $u_e x/\nu_e$
R_θ = Reynolds number, $u_e\theta/\nu_e$
St = Stanton number, see Eq. (31)
T = absolute temperature
u_w^* = friction velocity, $(\tau_w/\rho_w)^{1/2}$
u,v = x and y components of velocity
v_w^+ = dimensionless velocity ratio, v_w/u_w^*
x,y = rectangular coordinates
β = velocity-gradient parameter, $(2\xi/u_e)(du_e/d\xi)$
γ = intermittency factor
δ = boundary-layer thickness
δ^* = displacement thickness, $\int_0^\infty \left[1 - \frac{\rho u}{\rho_e u_e}\right] dy$
ϵ_m = kinematic eddy viscosity
ϵ_h = kinematic eddy conductivity
η = transformed y-coordinate
θ = momentum thickness, $\int_0^\infty \frac{\rho u}{\rho_e u_e} 1 - \frac{u}{u_e} dy$
μ = dynamic viscosity
ν = kinematic viscosity
ξ = transformed x coordinate
ρ = mass density
τ = shear stress
ψ = stream function

Subscripts

aw = adiabatic wall
i = inner region
e = outer edge of boundary layer
l = laminar flow
o = outer region
t = turbulent flow
w = wall
∞ = freestream conditions

Superscript

$'$ = differentiation with respect to η

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1. Introduction

SINCE 1965 the present author and his co-workers have been working on the development of a general boundary-layer method for calculating incompressible and compressible turbulent flows about two-dimensional and axisymmetric bodies. The method which uses the concepts of eddy-transport coefficients to eliminate the fluctuating quantities appearing in the governing equations, in principle, is similar to the ones used by Herring and Mellor[1] and by Patankar and Spalding.[2] The three methods differ chiefly in their eddy-transport expressions. In addition, the transformations used to stretch the coordinate normal to the flow direction and the numerical method used to solve the equations are considerably different.

So far the method has been applied to a number of turbulent flows, and the results are presented in several papers.[3–7] In almost all cases, the agreement between calculated results and experiment was remarkable. For example, for compressible adiabatic flows[6] the calculated velocity profiles, Mach profiles, and local skin-friction coefficients agreed quite well with experiments for a range of Mach numbers up to 5; the rms error in calculated local skin-friction values based on 43 experimental values obtained by the floating element technique was found to be 3.5%. It is important to note that these results were obtained by *one* eddy-viscosity formulation, by *one* turbulent Prandtl number formulation; and by a minimum of four empirical constants that were kept the same throughout the studies.

The present paper investigates the accuracy of the method for compressible turbulent boundary layers with heat and mass transfer. A number of turbulent flows are computed with this method and the results are compared with experiment. The calculated cases cover a Mach number range of 6.7.

The eddy-viscosity formulation used in this paper differs from the previous ones in that the eddy viscosity formulation of previous studies is generalized to handle compressible flows with heat and mass transfer.

2. Basic Equations

If the normal stress terms are neglected, the compressible turbulent boundary-layer equations for two-dimensional

flows can be written as
continuity

$$(\partial/\partial x)(\rho u) + (\partial/\partial y)(\rho v + \langle \rho' v' \rangle) = 0 \tag{1}$$

momentum

$$\rho u \frac{\partial u}{\partial x} + (\rho v + \langle \rho' v' \rangle) \frac{\partial u}{\partial y} = \rho_e u_e \frac{du_e}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y} - \rho \langle u' v' \rangle\right) \tag{2}$$

energy

$$\rho u \frac{\partial H}{\partial x} + (\rho v + \langle \rho' v' \rangle) \frac{\partial H}{\partial y} = \frac{\partial}{\partial y}\left[\frac{\mu}{Pr}\frac{\partial H}{\partial y} - \langle v' H' \rangle + \mu\left(1 - \frac{1}{Pr}\right) u \frac{\partial u}{\partial y}\right] \tag{3}$$

The boundary conditions are:
momentum

$$\begin{aligned} u(x,0) &= 0 \\ v(x,0) &= 0 \text{ or } v(x,0) = v_w(\text{mass transfer}) \\ \lim_{y \to \infty} u(x,y) &= u_e(x) \end{aligned} \tag{4}$$

energy

$$\begin{aligned} H(x,0) = H_w \quad &\text{or} \quad (\partial H/\partial y)(x,0) = (\partial H/\partial y)_w \\ \lim_{y \to \infty} H(x,y) &= H_e(x) \end{aligned} \tag{5}$$

3. Turbulent Transport Coefficient for Momentum and Heat Transport

In order to solve the system (1–5) it is necessary to relate the time-mean fluctuating quantities $-\langle \rho u' v' \rangle$ and $-\langle \rho v' H' \rangle$ to mean velocity and enthalpy distributions, respectively. Here, as in previous studies, we use eddy-viscosity (ϵ_m) and eddy-conductivity (ϵ_h) concepts. We define

$$-\langle \rho u' v' \rangle = \rho \epsilon_m \, \partial u/\partial y \tag{6}$$

$$-\langle \rho v' H' \rangle = \rho \epsilon_h \, \partial H/\partial y \tag{7}$$

and introduce a "turbulent Prandtl number," Pr_t:

$$Pr_t = \epsilon_m/\epsilon_h \tag{8}$$

Although these transport coefficients do not necessarily describe the microscopic details of a turbulent flow and do not provide basic information about the turbulence mechanism, they are very useful tools in engineering.

In the eddy-viscosity formulation we regard the turbulent boundary layer as a composite layer characterized by inner and outer regions. The existence of the two regions is due to the different response of fluid to shear and pressure gradient in each region. In the inner region, we use an eddy viscosity based on Prandtl's mixing-length theory,

$$(\epsilon_m)_i = l^2 |\partial u/\partial y| \tag{9}$$

where l, the mixing length, is given by $l = k_m y$ with $k_m = 0.4$. A modified expression for eddy viscosity is used to account for the viscous sublayer close to the wall. With this modification, suggested by Van Driest[8] and developed on the basis of a consideration of a Stokes-type flow, Eq. (9) becomes

$$(\epsilon_m)_i = (k_m y)^2 [1 - \exp(-y/A)]^2 |\partial u/\partial y| \tag{10}$$

where A is a damping length defined as $26\nu(\tau_w/\rho)^{-1/2}$.

In the outer region we use a constant eddy viscosity

$$(\epsilon_m)_0 = 0.0168 \left| \int_0^\infty (u_e - u) dy \right| \tag{11a}$$

modified by Klebanoff's intermittency factor[9] approximated by the following formula:

$$\gamma = [1 + 5.5(y/\delta)^6]^{-1} \tag{11b}$$

In the eddy-conductivity formulation, as in previous studies, we use the definition of turbulent Prandtl number and lump the eddy-conductivity term (7) into the turbulent Prandtl number given by Eq. (8), which is assumed to be constant and equal to 0.9.

4. Generalization of Inner Eddy-Viscosity Expression to Compressible Flows with Heat and Mass Transfer

The expression given by Eq. (10) was obtained by Van Driest for an incompressible flat-plate flow with no mass transfer. As it stands, it cannot be used for incompressible flows with pressure gradient or for flows with mass transfer. The former is quite obvious, since, for a flow with an adverse pressure gradient, τ_w may approach zero (flow separation). In such a case, the velocity profile will have a discontinuity. For this reason, in Ref. 10 the Van Driest expression was modified to account for incompressible flows with pressure gradient and mass transfer. In this section we will generalize the Van Driest expression to compressible flows with heat and mass transfer.

We consider Stokes flow, that is, a flow by an infinite flat plate that executes oscillations parallel to itself. The governing momentum equation for the flow is the one-dimensional nonsteady momentum equation. For a compressible flow it is given by

$$\rho \, \partial u/\partial t = (\partial/\partial y)(\mu \, \partial u/\partial y) \tag{12}$$

The above equation is subject to the boundary condition

$$u(0,t) = u_0 \cos\omega t \tag{13}$$

Introducing the transformation

$$dy = \mu dz \tag{14}$$

into Eq. (12), we get

$$\partial u/\partial t = (1/\rho\mu)(\partial^2 u/\partial z^2) \tag{15}$$

If $\rho\mu$ is taken to be an average value over the sublayer, Eq. (15) becomes

$$\partial u/\partial t = (1/\bar{\rho}\bar{\mu})(\partial^2 u/\partial z^2) \tag{16}$$

The solution of Eq. (16) subject to Eq. (13) is

$$u = u_0 e^{-mz} \cos(\omega t - mz) \tag{17}$$

where

$$m = [(\omega/2)\bar{\rho}\bar{\mu}]^{1/2} \tag{18}$$

From Eq. (17) we see that the amplitude of the motion diminishes with distance from the wall as a consequence of the factor $\exp(-mz)$. If we identify u as the fluctuation velocity u', we see that when the plate is fixed and the fluid oscillates relative to the plate, the maximum fluctuation velocity will be

$$u' = u_0'(1 - e^{-mz}) \tag{19}$$

where u_0' is the velocity fluctuation unaffected by the viscosity. Thus Eq. (19) shows that because of the viscous effects it is necessary to correct the velocity fluctuation by $(1 - e^{-mz})$. From the definition of Reynolds shear stress, we then can write

$$-\rho\langle u' v' \rangle = -\rho\langle u_0' v_0' \rangle (1 - e^{-mz})^2 \tag{20}$$

Since, according to Prandtl's mixing-length concept,

$$-\langle u_0' v_0' \rangle = l^2 (\partial u/\partial y)^2 \tag{21}$$

we see that Eq. (20) can then be written

$$-\rho\langle u'v'\rangle = l^2(1 - e^{-mz})^2(\partial u/\partial y)^2 \qquad (22)$$

If Eq. (18) is written in the form

$$m = [(\omega\bar{\nu}/2\bar{\nu})\overline{\rho\mu}]^{1/2} \qquad (23)$$

we see that the units of $(\omega\bar{\nu})^{1/2}$ are that of a velocity. We will take this velocity to be the sublayer friction velocity at the edge of the sublayer and define it by

$$(\omega\bar{\nu})^{1/2} = u_s^* = (\tau_s/\bar{\rho})^{1/2} \qquad (24)$$

Using Eq. (24), Eq. (23) can be written as

$$m = (u_s^*/\text{const})(\langle\overline{\rho\mu}\rangle/\bar{\nu})^{1/2} \qquad (25)$$

As in incompressible flows, we shall take the constant in Eq. (25) to be the same as the value in incompressible flows and write Eq. (25) as $m = (u_s^*/26)\bar{\rho}$. Taking μ in $\int dy/\mu$ to be $\bar{\mu}$, we can write the exponential term in Eq. (22 as

$$\exp(-mz) = \exp[-(u_s^*/26)(y/\bar{\nu})] \qquad (26)$$

The friction velocity based on the sublayer (u_s^*) can be determined from the momentum equation. Neglecting $\partial/\partial x(\rho u)$ in Eq. (1), we see that in the sublayer

$$\rho v + \langle\rho' v'\rangle = (\rho v)_w$$

Then, in the sublayer, Eq. (2) can be written as

$$\rho_w v_w\, \tau/\mu = -dp/dx + d\tau/dy \qquad (27)$$

where $\tau = \mu\, du/dy$. Since μ is a function of y, let us replace μ by its average value $\bar{\mu}$ in the sublayer. Then the solution of Eq. (27), with the boundary condition $\tau(0) = \tau_w$, and with $y_w^+ = 11.8$ as in incompressible flows, is given by

$$\frac{\tau_s}{\tau_w} = \left\{\frac{\bar{\mu}}{\mu_e}\left(\frac{\rho_e}{\rho_w}\right)^2 \frac{p^+}{v_w^+}\left[1 - \exp\left(11.8\,\frac{\mu_w}{\bar{\mu}}\,v_w^+\right)\right] + \exp\left(11.8\,\frac{\mu_w}{\bar{\mu}}\,v_w^+\right)\right\}^{1/2} = N^2 \qquad (28)$$

Equation (28) can also be written as

$$\tau_s/\bar{\rho} = (\tau_w/\rho_w)(\rho_w/\bar{\rho})N^2$$

or as

$$u_s^* = u_w^*(\rho_w/\bar{\rho})^{1/2}N \qquad (29)$$

Then the damping length A in Eq. (10) will be

$$A = 26\,\bar{\nu}/u_w^*(\rho/\bar{\rho}_w)^{1/2}\,1/N \qquad (30)$$

It should be noted that the aforementioned expression for the damping length has worked well for incompressible turbulent flows with heat and mass transfer[4] as well as for compressible adiabatic turbulent boundary layers.[6] In this paper we investigate its accuracy for compressible turbulent flows with heat and mass transfer.

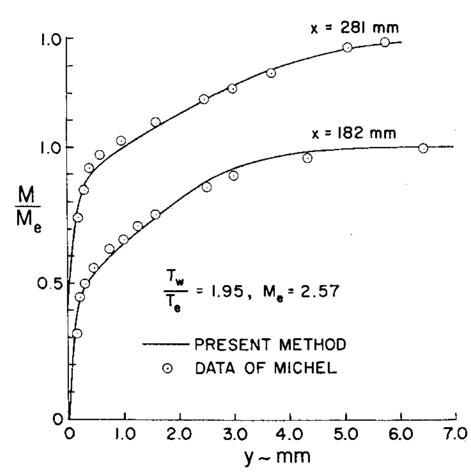

Fig. 1 Comparison of calculated and experimental Mach profiles for the boundary layer measured by Michel.

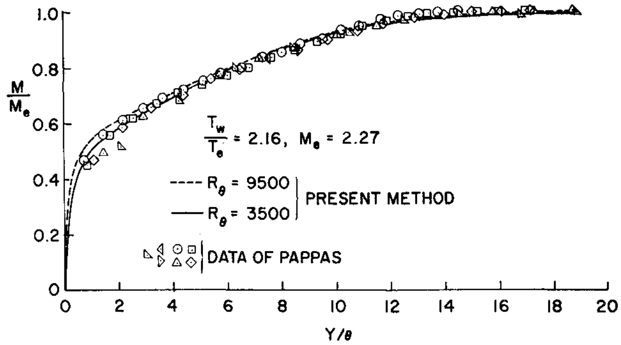

Fig. 2 Comparison of calculated and experimental Mach profiles for the boundary layer measured by Pappas.

5. Comparison of Calculated and Experimental Results

We present results of several computations with the present method and compare them with experiment. The numerical method is described in Ref. 11 and will not be repeated here. The calculations here are made by using the eddy-viscosity formulation given in Sec. 4 for a constant turbulent Prandtl number of 0.9. The average values of density $\bar{\rho}$ and viscosity $\bar{\mu}$ appearing in these equations are assumed to be given by their local values, that is, $\bar{\rho} = \rho$ and $\bar{\mu} = \mu$.

Figure 1 shows comparisons of calculated and experimental Mach profiles for the boundary layer measured by Michel at a Mach number of 2.57.[12] The calculations were made by starting the flow to be compressible laminar at $\xi = 0$, and specifying the flow to be turbulent at the next ξ station for $T_w/T_e = 1.95$, which was assumed constant along the plate. The computations were then carried downstream until the experimental R_x was obtained.

Figures 2 and 3 show the results for the boundary layer measured by Pappas.[13] The calculations were made for $M_e = 2.27$ and $T_w/T_e = 2.16$. Again the calculations were made by starting the flow to be compressible laminar at the leading edge, and specifying the flow to be turbulent at the next ξ station. The experimental momentum Reynolds number varied between 3000 and 10,000. For this reason, the calculated Mach profiles, shown in Fig. 2, were compared with the experimental data for $R_\theta = 3500$ and 9500. The agreement is good and the calculations account for the R_θ effect.

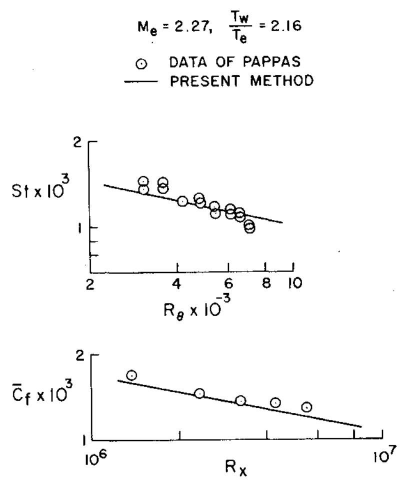

Fig. 3 Comparison of calculated and experimental results for the boundary layer measured by Pappas.

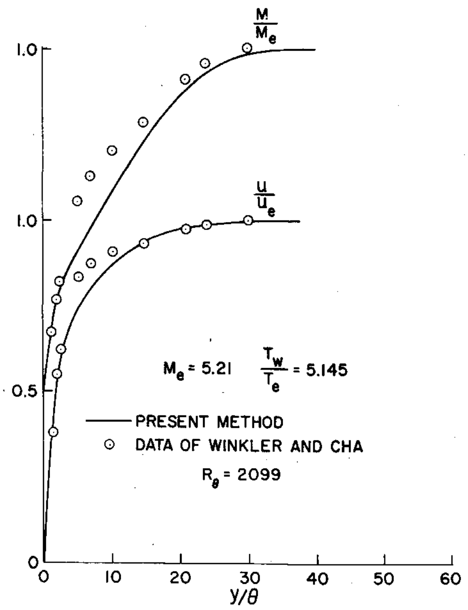

Fig. 4 Comparison of calculated and experimental velocity and Mach profiles for the boundary layer measured by Winkler and Cha at $R_\theta = 2099$.

Figure 3 shows a comparison of calculated and experimental values of local Stanton number St, defined as

$$St = -\dot{q}_w/\rho_e u_e(H_{aw} - H_w) \tag{31}$$

and values of local skin-friction coefficient c_f, defined as

$$c_f = 2\tau_w/\rho_e u_e^2 \tag{32}$$

In the calculations, the adiabatic wall enthalpy H_{aw} was obtained by repeating the calculations for an adiabatic flow.

Figures 4–6 show comparisons of calculated and experimental velocity profiles, Mach profiles, local skin-friction coefficient and local Stanton number for the boundary layer measured by Winkler and Cha.[14] The calculations were made for $M_e = 5.21$ and $T_w/T_e = 5.145$. The results show poor agreement in velocity and Mach profiles at low R_θ values. However, at higher R_θ values, the agreement is quite satisfactory. The calculated local skin-friction values and Stanton number values are also in good agreement with experiment.

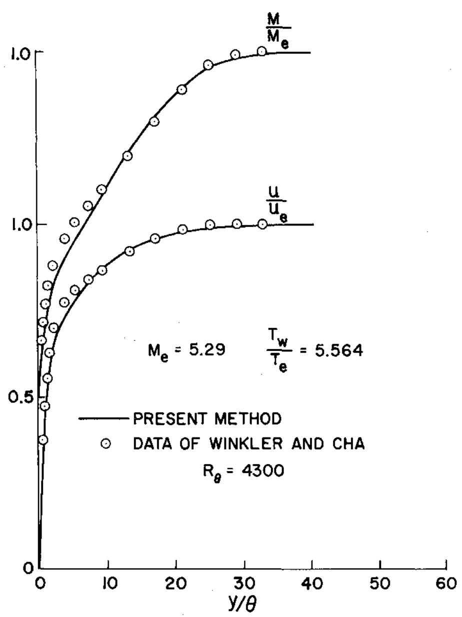

Fig. 5 Comparison of calculated and experimental velocity and Mach profiles for the boundary layer measured by Winkler and Cha at $R_\theta = 4300$.

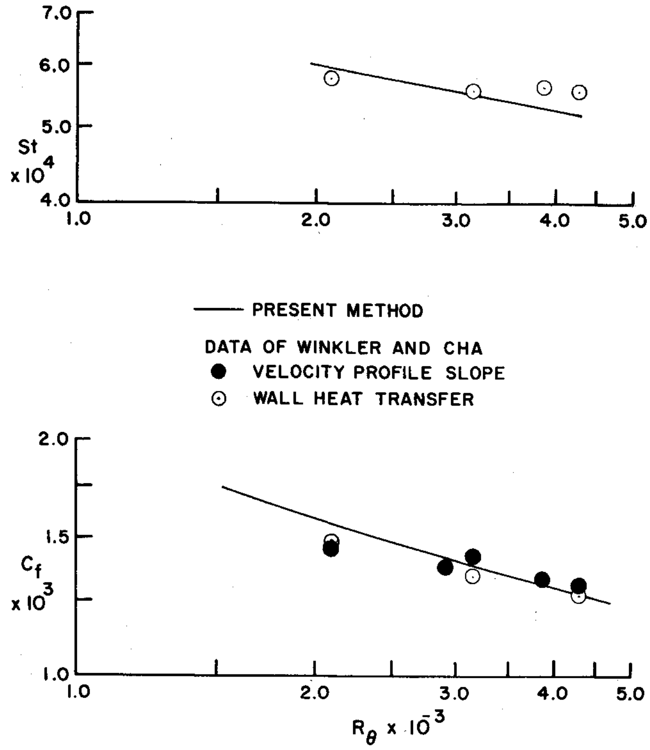

Fig. 6 Comparison of calculated and experimental results for the boundary layer measured by Winkler and Cha.

Figure 7 shows the results for an accelerating flow measured by Pasiuk et al.[15] Calculations were started by assuming a constant heat flux flat-plate flow that matched the experimental momentum-thickness value at $x = 0.94$ ft. Then the experimental Mach number distribution was used to compute the rest of the flow for constant heat flux. The edge Mach number varied from $M_e = 1.69$ at $x = 0.94$ ft to $M_e = 2.97$ at $x = 3.03$ ft. Except at one ξ station, the calculated profiles are in good agreement with experiment.

Figures 8–10 show the results for the boundary layer measured by Danberg.[16] The calculations were made for a Mach number of 6.7 and include flows with and without mass transfer.

Figure 8 shows a comparison of calculated and experimental R_θ, H, c_f and St values for no mass transfer. The calculations were made for $T_w/T_e = 4.48$. Figures 9 and 10 show a comparison of results for mass transfer. The calculations were made for $T_w/T_e = 4.17$ and $F = 9.09 \times 10^{-4}$.

Figures 11 and 12 show the results for the adiabatic boundary layer measured by Jeromin[17] at $M_e = 2.5$ for $F = 0.421 \times 10^{-3}$.

In Ref. 17 Jeromin compared his experimental static temperature distribution with the temperature formula

$$T = T_w - (T_w - T_{aw})(u/u_\infty) - (T_r - T_\infty)(u/u_\infty)^2 \tag{33}$$

which was proposed by Spence, Crocco, and Van Driest. He observed that the agreement between experiment and the above equation for various injection rates was satisfactory. He used the above formula for analyzing most of his bound-

Table 1 Comparison of calculated and experimental local skin-friction coefficients for the data of Squire[19]

F	$(c_f)_{\text{cal}} \times 10^3$	$(c_f)_{\text{exp}} \times 10^3$	
0	2.02	2.03	(2.08)
1.3	1.50	1.40	(1.52)

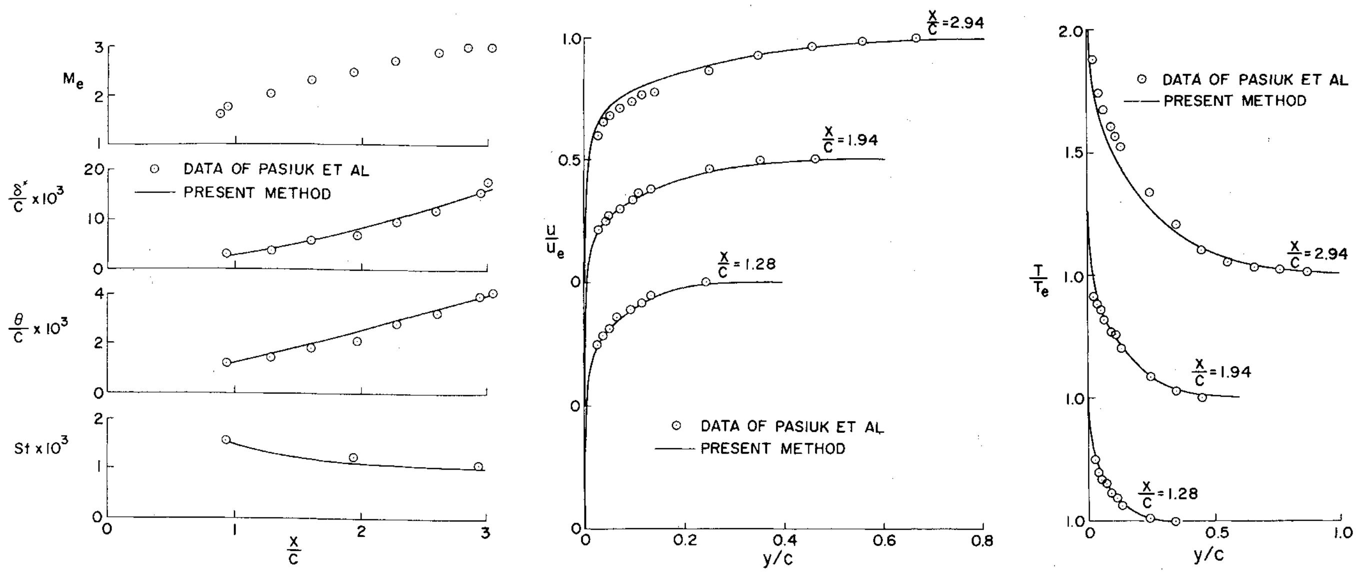

Fig. 7 Comparison of calculated and experimental results for the accelerating flow measured by Pasiuk et al.

ary-layer profiles, instead of measuring the temperature profile for each. However, the difficulty in the application of the temperature formula is the lack of accurate recovery factor for flows with mass transfer. For that reason, he used Bartle and Leadon's data[18] for $M_e = 2$ and 3.2.

Figure 12 shows a comparison of calculated static-temperature distribution for $F = 0.421 \times 10^{-3}$ with those obtained from the temperature formula. In the application of the temperature formula, the recovery factor was obtained from Bartle and Leadon's data (as Jeromin did), as well as from the present method. The results seem to indicate a better agreement when the calculated recovery factor is used.

Figures 13 and 14 show a comparison of calculated and experimental velocity profiles for the boundary layer measured by Squire[19] at a Mach number of 1.80. The calculations were made with and without mass transfer. Although for zero mass transfer the agreement in velocity profiles is good, with mass transfer they are not. On the other hand, the agreement in local skin friction is good for both cases (see Table 1). Experimental skin-friction values were obtained from the momentum integral equation. The values in parenthesis show the value of c_f obtained by ignoring the pressure gradient term in the momentum integral equation.

The present method was also used to compute the boundary layer measured by Dershin and Leonard.[20] The measurements were made at $M_e = 3.2$. The skin-friction values were obtained by a skin-friction balance that permitted mass in-

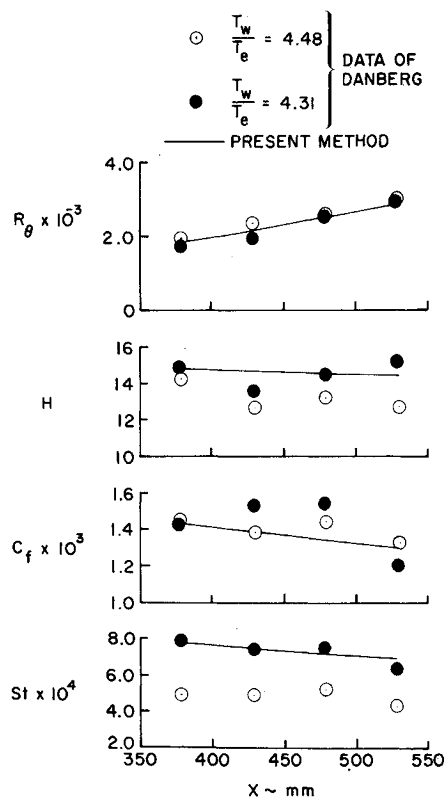

Fig. 8 Comparison of results for the boundary layer measured by Danberg. Calculations were made for $T_w/T_e = 4.48$.

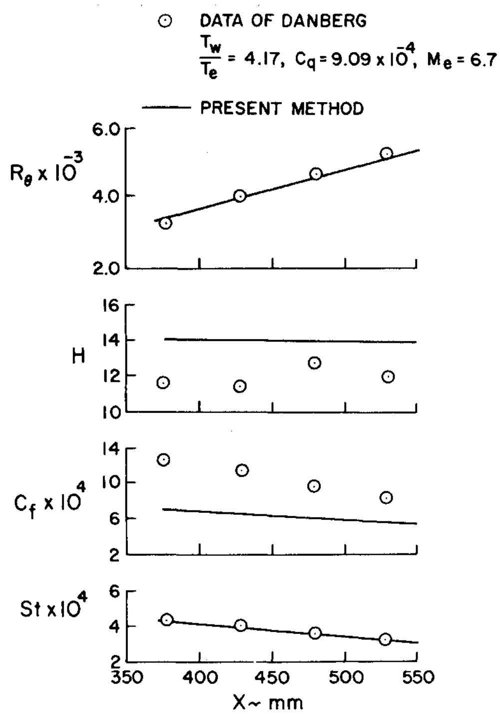

Fig. 9 Comparison of results for the boundary layer measured by Danberg.

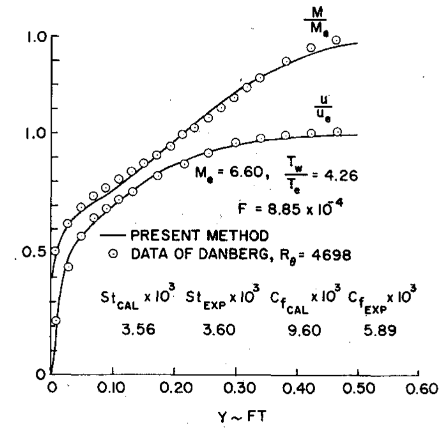

Fig. 10 Comparison of calculated and experimental Mach profiles, velocity profiles, local skin friction and Stanton number values for the boundary layer measured by Danberg.

jection through its friction surface. For the test conditions the R_θ was about 33,000. The authors state that for zero blowing the local skin-friction coefficient is about 0.0011, according to Deissler and Loeffler's theoretical prediction.[21]

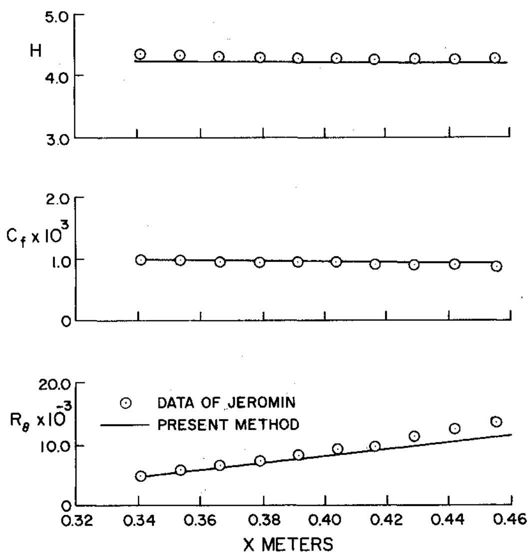

Fig. 11 Comparison of calculated and experimental results for the boundary layer measured by Jeromin; $M_e = 2.5$, $F = 0.421 \times 10^{-3}$.

Dershin and Leonard's measured value was 0.0009. The value calculated by the present method is 0.00128. However, it is believed that this value is probably more accurate than

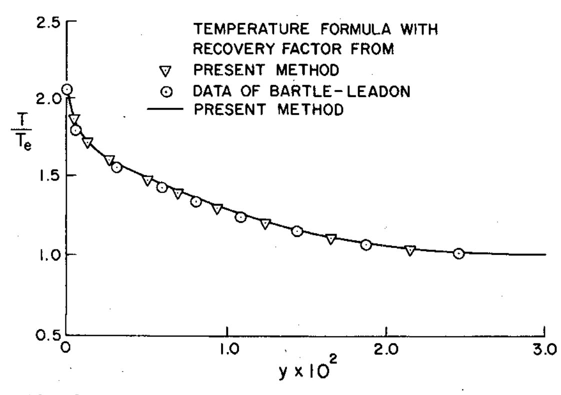

Fig. 12 Comparison of temperature distributions for Jeromin's data; $M_e = 2.5$, $F = 0.421 \times 10^{-3}$.

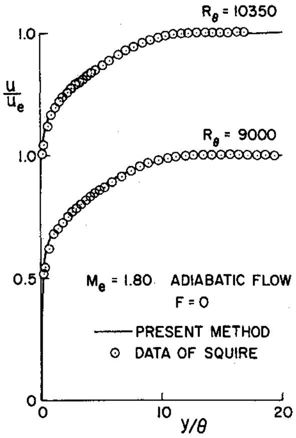

Fig. 13 Comparison of calculated and experimental velocity profiles for the boundary layer measured by Squire.

the others, since a previous study, reported in Ref. 6, showed that the present method was very accurate for adiabatic turbulent flows.

Figure 15 shows the ratio of c_f/c_{f_0} vs $2F/c_{f_0}$ for the data of Dershin and Leonard. The theoretical predictions of Rubesin[22] and the prediction of the present method are shown in the same figure. It is important to note that although experimental c_f values alone do not agree with the theoretical predictions, the ratio of the experimental values does, raising the question of an "error" in measurements.

6. Concluding Remarks

Based on the results obtained in the study and on the results obtained in previous studies,[3–7] we can make the following observations on the accuracy of the present method.

1) The method is quite accurate for incompressible turbulent boundary layers with heat and mass transfer. This applies for both two-dimensional and axisymmetric flows including transverse-curvature effects. The deficiency of the method is at flow conditions near separation. Although the method predicts flow separation quite accurately,[23] the predicted velocity profiles and consequently, the temperature profiles close to separation, are not satisfactory. Further work needs to be done.

2) The method is quite accurate for compressible adiabatic turbulent flows with no mass transfer for Mach numbers up to 5 (Ref. 6). In the case of compressible flows with heat and mass transfer, the results, as presented in this paper, are satisfactory; but they are by no means as good as those in adiabatic cases. At high Reynolds numbers the agreement gets better. However, with increasing Mach number and decreasing Reynolds number, the calculated results deviate from the experimental data. Furthermore, the agreement, especially in temperature profiles, decreases with increasing heat-transfer

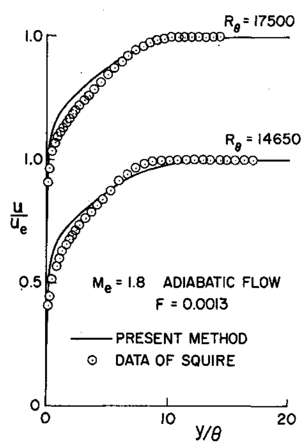

Fig. 14 Comparison of calculated and experimental velocity profiles for the boundary layer measured by Squire.

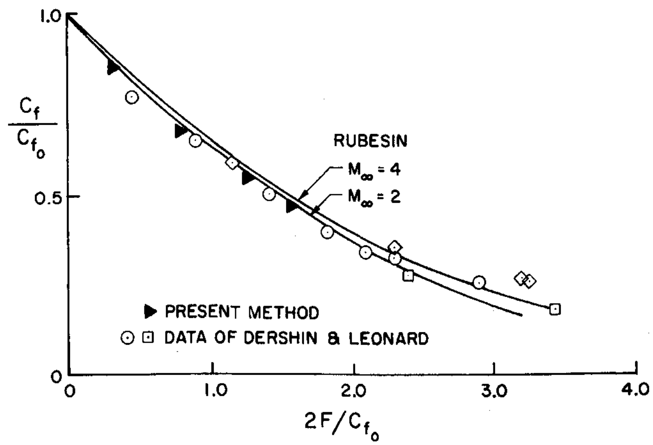

Fig. 15 Comparison of calculated and experimental results for Dershin and Leonard's data; M_e = 3.2, R_θ = 33,000.

rate and Mach number. Consequently, further work needs to be done at low Reynolds numbers and high heat-transfer rates for high Mach number flows.

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